

Maximizing Data Rate for Multiway Relay Channels with Pairwise Transmission Strategy

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Abstract

In a multiway relay channel (MWRC), pairwise transmission strategy can be used to reduce the computational complexity at the relay and the users without sacrificing the data rate, significantly. The performance of such pairwise strategies, however, is affected by the way that the users are paired to transmit. In this paper, we study the effect of pairing on the common rate and sum rate of an MWRC with functional-decode-forward (FDF) relaying strategy where users experience asymmetric channel conditions. To this end, we first develop a graphical model for an MWRC with pairwise transmission strategy. Using this model, we then find the maximum achievable common rate and sum rate as well as the user pairings that achieve these rates. This marks the ultimate performance of FDF relaying in an MWRC setup. Further, we show that the rate enhancement achieved through the optimal user pairing becomes less pronounced at higher SNRs. Using computer simulations, the performance of the optimal pairing is compared with those of other proposed pairings in the literature.

Index Terms

Multiway relay channels, functional-decode-forward, pairwise relaying, common rate, sum rate.

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A portion of this work was presented in ISIT 2014.

I. INTRODUCTION

A multiway relay channel (MWRC) [1] is an extension of a two-way relay channel [2]–[6] in which $N \geq 2$ users intend to share their data. This could be partial data sharing, where each user shares its data with not all but a subset of other users, or full data sharing when each user share its data with all other users. It is common to assume that no direct link is present between the users and a relay assists them to communicate their messages. Conference calls, file sharing, and multi-player gaming [7], [8] are potential applications of MWRCs. Different from conventional cooperative schemes, each user serves as both data source and data destination in an MWRC. This means that we have simultaneous data flows in different directions necessitating the design of customized transmission strategies for MWRCs.

Pairwise transmission [1], [7], [9]–[11], also known as *pairwise network coding*, is one of the main transmission strategies proposed for MWRCs. To accomplish full data exchange based on pairwise transmission, a set of pairs, representing the users' transmission schedule, is defined. Every two users within a pair simultaneously send their data to the relay in an uplink phase. In *functional-decode-forward* (FDF) [10], the relay directly decodes a function of the two received messages. Following the uplink phase, the relay broadcasts the function of the two users' messages to all users [1] in a downlink phase. The uplink and downlink transmissions continue until each user is capable of decoding all other users' messages. It is worth mentioning that pairwise transmission has a low decoding complexity, while offering interesting capacity-achieving properties in various MWRC setups [10], [11]. For instance, it has been shown that pairwise transmission along with rate splitting and joint source-channel decoding achieves the capacity region of MWRC over finite fields [11].

In an MWRC with pairwise transmission, the way that users are paired for transmission,

referred to as *users' pairing*, directly affects the achievable data rates of the system [7]. That said, the effect of pairing on the common rate (the rate that any user can reliably transmit its data with this rate to all other users) of an MWRC has been the subject of various studies. Considering different constraints on the relay transmit power, authors in [12] have shown that their pairing strategy maximizes the common rate for an MWRC with FDF relaying where each user's transmitted signal can depend on both its own message as well as its previously received signals. In [7], authors have found the optimal pairing to maximize the achievable common rate of the users for an MWRC with asymmetric Gaussian channels under the assumption that each user transmits in at most two uplink phases. In [13], an opportunistic approach for finding the pairing in a pairwise transmission for MWRC with compute-and-forward relaying has been proposed. Further, [14] has considered a pairing in which the user with the highest SNR is paired with all other users and the common rate and sum rate of the system have been investigated for various channel configurations.

In this work, we seek optimal pairings to maximize the common rate and sum rate for FDF relaying. Thus, the ultimate rate performance achieved by FDF is determined. To this end, we first introduce a graph-based modeling for the data transmission in a pairwise MWRC. Using this model, we then find the necessary and sufficient conditions for a pairing to be feasible, i.e., each user is able to retrieve the data of all other users. Using this condition, we then discuss that there exist N^{N-2} distinct feasible pairings in the system. Thus, finding the optimal pairing through brute-force search becomes extremely expensive as N increases. That said, it is desired to analytically find the optimal pairings. To address this, we use the developed graph-based model to analytically find common rate and sum rate maximizing pairings.

The rest of the paper is organized as follows: In Section II, we describe the system model.

In Section III, we introduce a graphical representation of the transmission pairing. Then, we describe the sum rate and the common rate maximization problems that we want to solve. Our proposed graphical model is used to find the solutions to these problems in Section IV. We compare the performance of our proposed pairings with those of other transmission strategies in the literature via simulations in Section V. Finally, Section VI concludes the paper¹.

II. SYSTEM MODEL

We consider an MWRC in which N single-antenna users, namely U_1, U_2, \dots, U_N , perform full data exchange meaning that each user wants to send/receive data to/from all other users. It is assumed that users cannot communicate directly, thus, a single-antenna relay \mathcal{R} assists them to share their data (Fig. 1). Here, all U_i 's transmit their (encoded) message X_i 's with equal power P and P_r is the transmit power of the relay. The channel from U_i to \mathcal{R} is assumed to be reciprocal and slow-fading so that the channel gain h_i remains unchanged during the data exchange between the users. Further, additive white Gaussian noise (AWGN) with variance σ^2 is assumed at the relay and users.

The uplink signal to noise ratio (SNR) for user U_i , namely γ_i , is defined as $\gamma_i \triangleq \frac{P|h_i|^2}{\sigma^2}$. Without loss of generality, we assume that $|h_1| \leq |h_2| \leq \dots \leq |h_N|$, and thus²

$$\gamma_N \geq \gamma_{N-1} \geq \dots \geq \gamma_1 > 0. \quad (1)$$

¹This work was partially presented at the IEEE International Symposium on Information Theory, Honolulu, HI, USA, 2014 [15]. In [15], the results for optimal pairing has been presented without proofs. The current work also extends the sum rate results of [15] to cases that some users are listeners only and do not participate in data sharing.

²Note that for any given MWRC with N users, we can always label the user with the worst SNR (poorest channel) as U_1 , the second worst user as U_2 , and so on. Thus, for any MWRC, we can always have $|h_1| \leq |h_2| \leq \dots \leq |h_N|$ via a simple relabeling of the users.

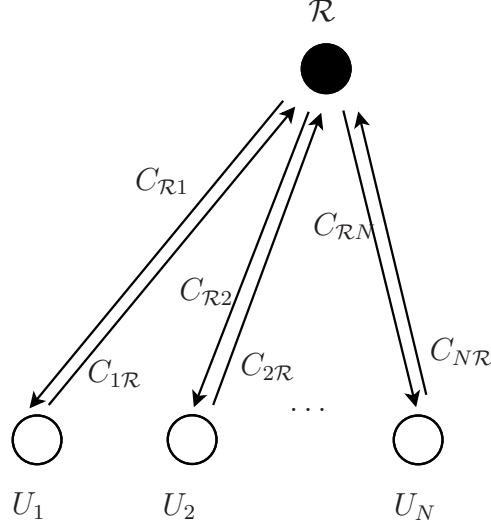


Fig. 1. An MWRC with N users.

Similarly, a user downlink SNR is defined as $\Gamma_i \triangleq \frac{P_r |h_i|^2}{\sigma^2}$. We denote the minimum of downlink SNRs by $\Gamma_d = \min_i \{\Gamma_i\}$. Note that since $\min_i |h_i| = |h_1|$, $\Gamma_d = \Gamma_1$.

A. Pairwise Transmission Strategy

In a pairwise transmission scheme, users are grouped into $M = N - 1$ pairs³. These pairs are not necessarily disjoint meaning that a specific user can appear in more than one pair. However, all messages sent by a user in a round of communication are identical. This is necessary to ensure successful decoding at the users as discussed later. Such a set of pairs is called a *pairing* of the users and is denoted by an M -tuple $O = (\{u_{11}, u_{12}\}, \dots, \{u_{M1}, u_{M2}\})$ where $u_{\ell 1}$ and $u_{\ell 2} \in \{U_1, U_2, \dots, U_N\}$ for any $\ell \in \{1, \dots, M\}$.

³Note that here, any of the users wants to multicast its data to all other users. Here, the term pairwise refers to the transmission strategy in a MWRC and should not be confused with multi-user or multi-pair two way relaying [16]–[19] where several unicast message exchanges happen between the users.

Now that we have defined O , we explain how the users share their data over a data exchange round, i.e. M uplink phases and M downlink phases. In the ℓ th uplink phase, $\ell \in \{1, \dots, M\}$, the users associated with the ℓ th pair of the pairing simultaneously send their data to the relay. Let us call the users in this pair by U_i and U_j that apply a coding scheme defined over a field \mathbb{F} to produce their coded messages (vectors) X_i and X_j respectively. The uplink phase is then followed by a downlink phase where the relay broadcasts the sum of the two messages received in the last uplink phase to the users. That is, the relay broadcasts $X_i \oplus X_j$ in the downlink phase where \oplus refers to the element-wise summation of X_i and X_j over \mathbb{F} . The discussion on how the relay forms its transmit messages is discussed later.

These pairwise transmissions continue until the last pair of the pairing. After the last downlink transmission, each user has a set of M equations where each equation is a linear combination of two users' messages. By having the knowledge of self message, U_i attempts to solve its received set of equations successfully. Note that the transmit power of the users is fixed for during each uplink phase of an each exchange round. For more details please see [10]. If the system of M equations at each user is solvable, we say that the corresponding pairing is *feasible*. The notion of feasibility ensures that each user can find all other messages.

A clear advantage of a pairwise relaying over joint multi-user decoding (e.g. full decode and forward), is its lower complexity. While the decoding complexity of joint decoding grows exponentially with the number of users [20], [21], in a pairwise system the complexity grows linearly with N (or as constant when normalized by the number of users). This is because each user decodes the message of other users one by one. Lower decoding complexity also benefits the relay where it only needs to deal with the message of only two users at a time regardless of N . One should note that for the pairwise system to work, each user has to know the pairing

schedule. This can be broadcast to the users by the relay.

B. Achievable Data Rates

Users employ channel codes to protect their data against the noise. For the assumed MWRC with N users, a $(2^{nR_1}, 2^{nR_2}, \dots, 2^{nR_N}, n)$ code consists of the following four components:

- Users' messages: They are represented by N sets of integers $\mathcal{W}_i = \{1, 2, \dots, 2^{nR_i}\}$ for $i = 1, \dots, N$. Each set \mathcal{W}_i represents the U_i 's original messages.
- Users' encoding functions: An encoding function $f_i(\cdot)$ is assigned to U_i that takes a $W_i \in \mathcal{W}_i$ and forms X_i as $X_i = f_i(W_i)$.
- Relay's encoding function: The relay's transmit message at the ℓ th downlink phase, X_r^ℓ , is formed by the encoding function $f_r(\cdot)$ as $X_r^\ell = f_r(Y_r^\ell)$ where Y_r^ℓ is the relay's received signal from the ℓ th uplink phase.
- Users' decoding functions: The decoding function at U_i uses the received signals from all downlink phases at U_i as well as the knowledge of self message to decode the data of all other users. In other words,

$$(\hat{W}_1, \dots, \hat{W}_N) = g_i(Y_i^1, \dots, Y_i^M, W_i) \quad (2)$$

where Y_i^ℓ is the received signal at U_i in the ℓ th downlink phase and \hat{W}_j is the estimate of W_j .

For the aforementioned code, the average probability of error is

$$P_e^n = \Pr \bigcup_{i=\{1, \dots, N\}} \{g_i(Y_i^1, \dots, Y_i^M, W_i) \neq (W_1, \dots, W_N)\}. \quad (3)$$

Now, a rate tuple (R_1, R_2, \dots, R_N) is said to be achievable if there exists a code where $P_e^n \rightarrow 0$ as n goes to infinity. In other words, any U_i can reliably (with arbitrarily small probability

of error) transmit its data to all other users with rate R_i after each round's M uplink and downlink phases (i.e., a complete round of full data exchange). Knowing that (R_1, R_2, \dots, R_N) is achievable, all users can reliably share their data with a common rate R_c where

$$R_c \triangleq \min_i R_i. \quad (4)$$

Also, the sum rate R_s is defined as

$$R_s \triangleq \sum_{i=1}^N R_i. \quad (5)$$

C. FDF Relaying

Assume that at the ℓ th uplink phase $u_{\ell 1} = U_i$ and $u_{\ell 2} = U_j$ are paired and transmit their data to the relay. As a result, the received signal at the relay is

$$Y_{r,\ell} = h_i X_i + h_j X_j + Z_r \quad (6)$$

where Z_r is the AWGN at the relay. After receiving $Y_{r,\ell}$, the relay forms its message $X_{r,l} = X_i \oplus X_j$ to be transmitted to all users in the ℓ th downlink phase. To form $X_i \oplus X_j$, the relay uses nested lattice codes [3], [22] at the users for a more efficient decoding at the relay. The basic notion of this technique is that by using lattice codes, the relay is capable of directly decoding the summation of the received messages. It means that it decodes $X_i \oplus X_j$ rather than separately decoding X_i and X_j . For more details on FDF, see [12], [22]. After the relay's transmission in the ℓ th downlink phase, the received signal at an arbitrary user k is

$$Y_{k,\ell} = h_k X_{r,l} + Z_k \quad (7)$$

where Z_k is the AWGN at U_k .

Having the above transmission model and using the results in [7], [22], The achievable rates of U_i and U_j , denoted by R_i and R_j respectively, are limited by the following achievable upper

bounds

$$R_i \leq \max \left\{ 0, \frac{1}{2M} \log_2 \left(\frac{\gamma_i}{\gamma_i + \gamma_j} + \gamma_i \right) \right\}, \quad (8)$$

$$R_j \leq \max \left\{ 0, \frac{1}{2M} \log_2 \left(\frac{\gamma_j}{\gamma_i + \gamma_j} + \gamma_j \right) \right\}. \quad (9)$$

In addition, the transmission rates of the users may be limited by the downlink phase. More specifically, the transmit rate of any U_i is bounded as follows

$$R_i \leq \frac{1}{2M} \log_2 (1 + \Gamma_d). \quad (10)$$

Following the above, the overall upper bound on R_i is now found by taking the minimum of all uplink upper bounds on R_i and its downlink upper bound.

As seen from (8), R_i , and as a consequence the common rate and sum rate of the system, are functions of $\frac{\gamma_i}{\gamma_i + \gamma_j}$. On the other hand, since the choice of the users' pairing affects $\frac{\gamma_i}{\gamma_i + \gamma_j}$, both common rate and sum rate are affected by the users' pairing. This motivates us to seek pairings that maximize R_c and R_s . In the following section, we explain the problem of finding such pairings in more detail.

III. PROBLEM DEFINITION

In this section, we first introduce the concept of client graph which is a graphical description of users' pairing. Then, we define the problems that we study in this work. As we assume that all channel gains remain unchanged during each round of full data exchange, it does not matter which of the available M uplink phases are allocated to a pair. That is, while the way that the pairs are formed affects the common and sum rate, allocation of the uplink phases to the pairs is irrelevant to the data rate. Thus, instead of using an M -tuple $O = (\{u_{11}, u_{12}\}, \dots, \{u_{M1}, u_{M2}\})$ to denote a pairing, we use a set representation as $O = \{\{u_{11}, u_{12}\}, \dots, \{u_{M1}, u_{M2}\}\}$ in the rest of the paper.

A. Client Graph

An undirected graph G is an ordered pair $G = (V, E)$ comprising a set $V = \{v_1, v_2, \dots, v_K\}$ of vertices together with a set E of edges. For simplicity, if $\{v_i, v_j\} \in E$, we say $v_i v_j \in E$. If $v_i v_j \in E$, we say v_j is *adjacent* to v_i . The set of adjacent vertices of v_i , denoted by A_i^G , is called the set of neighbors of v_i . Also, the *degree* of node v_i is $\deg(v_i) = |A_i^G|$. The adjacency matrix of G , denoted by $\mathcal{A} = (a_{ij})$, is a $K \times K$ matrix in which $a_{ij} = 1$ if and only if (iff) $v_i v_j \in E$; otherwise $a_{ij} = 0$. A *path* in G is a sequence of consecutive edges that connects a sequence of vertices. G is called connected if there is at least one path between every pair of its vertices. A non-empty path with the same endpoints is called a *cycle*.

For a given pairwise pairing O , we define a *client graph* $G_O(V, E)$ where $V = \{v_1, v_2, \dots, v_N\}$ is the set of vertices. There is a vertex v_i in V corresponding to each user U_i . There exists an edge $e = v_i v_j \in E$ iff $\{U_i, U_j\} \in O$. Note that there is a one-to-one correspondence between all possible client graphs and all possible pairwise pairings. As an example, Figure 2 depicts the client graph associated with pairing $O_{R_c} = \{\{U_1, U_2\}, \{U_2, U_3\}, \{U_3, U_4\}, \dots, \{U_{N-1}, U_N\}\}$.

Theorem 1. *A pairing with $M = N - 1$ pairs is feasible iff the corresponding client graph is a tree⁴.*

Proof: For the forward direction, note that if the client graph is not a tree, then it has $k > 1$ components. That is, the corresponding system of linear equations consists of k uncoupled systems of equations. This contradicts the feasibility of the pairing. For the backward direction, we use the fact that if the client graph is a tree, then there is exactly one path $P_{i,j}$ between any pair of nodes v_i and v_j . Assume that $P_{i,j} = \{v_i v_{i_1}, v_{i_1} v_{i_2}, \dots, v_{i_{n-1}} v_j\}$. The equations corresponding

⁴In graph theory, a tree refers to a graph that does not have any cycle.

to the edges in this path are:

$$\begin{aligned}
X_i \oplus X_{i_1} &= X_r^{m_1} \\
X_{i_1} \oplus X_{i_2} &= X_r^{m_2} \\
&\vdots \\
X_{i_{n-1}} \oplus X_j &= X_r^{m_n}
\end{aligned} \tag{11}$$

in which $X_r^{m_k}$ represents the relay message at the corresponding downlink. Manipulating this system of equations, we wind up with

$$X_i \oplus (-1)^{n-1} X_j = \bigoplus_{k=1}^n (-1)^{k-1} X_r^{m_k}. \tag{12}$$

Knowing its own data, U_i can decode X_j for all $j \neq i$ according to (12). Thus, if the client graph is a tree, the corresponding pairing is feasible. ■

In the following, we use the terms *client tree* and *client graph*, interchangeably. Further, we denote the maximum achievable common rate and sum rate for a client graph G_O by $R_c(G_O)$ and $R_s(G_O)$, respectively.

B. Common Rate and Sum Rate Maximization

In this paper, we focus on two problems related to the data rate performance of the considered MWRC. First, we consider *common rate maximization problem* meaning that we are interested in finding a pairing that maximizes R_c . More formally, if we denote the set of all feasible pairings with \mathcal{O} , the optimal pairing is defined as

$$O_{R_c} = \operatorname{argmax}_{O \in \mathcal{O}} R_c(G_O) \tag{13}$$

The second problem is *sum rate maximization problem* where we want to find the pairing that maximizes the sum rate of the considered MWRC. To address this problem, we assume that all

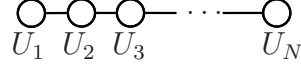


Fig. 2. Client tree that maximizes $R_c(G_O)$ for a pairwise MWRC with FDF relaying.

users want to participate in each round of full data exchange and transmit data with a non-zero rate. This can be interpreted as a level of fairness [14] between all users such that we do not push any of them to stop transmitting. However, we will discuss how this can be extended to a more general setup where some users may stay silent in an exchange round. Furthermore, in the sum rate maximization problem, it is assumed that achievable rates are not limited by the downlink phase and we only focus on the effect of the pairing on the achievable rates of uplink. To be more specific, we are interested to find an optimal pairing such that

$$O_{R_s} = \arg \max_{O \in \mathcal{O}} R_s(G_O). \quad (14)$$

In order to solve a common (sum) rate maximization problem, we need to find a client graph G_O with greatest $R_c(G_O)$ ($R_s(G_O)$) among all client trees. One way is to search over all possible client trees and find the one that maximizes $R_c(G_O)$ ($R_s(G_O)$). According to Cayley's formula [23], this approach needs searching over N^{N-2} client trees which is impractical even if the number of users is not very large. This motivates us to develop efficient solutions for finding the optimal client trees without going through such tedious searches.

IV. PROBLEM SOLUTION

In this section, we provide solutions to the common rate and sum rate maximization problems as defined by (13) and (14) for FDF relaying. We emphasize that the optimality considered in this section is limited to finding the best choice for pairing in a pairwise transmission strategy.

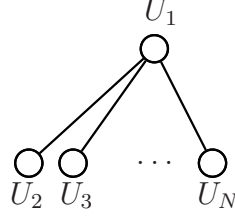


Fig. 3. Client tree that maximizes $R_s(G_O)$ for a pairwise MWRC with FDF relaying subject to the weakened upper bound given by (17).

A. Common Rate Maximization

Considering (8), we find the pairing that achieves the maximum $R_c(G_O)$ for FDF relaying.

Theorem 2 gives the optimal pairing for this scenario.

Theorem 2. *The pairing given by*

$$O_{R_c} = \{\{U_1, U_2\}, \{U_2, U_3\}, \{U_3, U_4\}, \dots, \{U_{N-1}, U_N\}\} \quad (15)$$

achieves the maximum common rate in an MWRC with FDF relaying and the maximum achievable common rate is

$$R_c(G_O) = \frac{1}{2(N-1)} \min_{i \in \{1, \dots, N\}} + \left\{ \log_2 \left(\gamma_i + \frac{\gamma_i}{\gamma_i + \gamma_{i+1}} \right), \log_2 (1 + \Gamma_d) \right\}. \quad (16)$$

in which $\min^+ A = \max\{0, \min A\}$.

Proof: See Appendix A. ■

Fig. 2 illustrates the client graph to achieve maximum R_c with FDF relaying. Using the results in [7], it can be shown that asymptotically, as uplink SNRs increase, the performance of random pairing achieves the performance of optimal pairing.

B. Sum Rate Maximization

To find a pairing with maximum sum rate, we make two assumptions. First, the downlink does not limit the rate. Second, the transmit SNRs are not too low. More specifically, for any i and j , $\frac{\gamma_i}{\gamma_i + \gamma_j} + \gamma_i \geq 1$. This assumption generally holds in most practical setups where the signal power is stronger than the noise power. To this end, the upper bound on achievable rate of U_i , when it is paired with U_j , is given by:

$$R_i \leq \frac{1}{2(N-1)} \log_2 \left(\frac{\gamma_i}{\gamma_i + \gamma_j} + \gamma_i \right). \quad (17)$$

Now, the optimal pairing is given by the following theorem:

Theorem 3. *The pairing*

$$O_{R_s} = \{\{U_2, U_1\}, \{U_3, U_1\}, \dots, \{U_N, U_1\}\} \quad (18)$$

is the optimal pairing for FDF relaying subject to (17). Moreover, the maximum achievable sum rate for this pairing is:

$$R_s(G_O) = \frac{1}{2(N-1)} \log_2 \left(\left(\gamma_1 + \frac{\gamma_1}{\gamma_1 + \gamma_N} \right) \times \prod_{i=2}^N \left(\frac{\gamma_i}{\gamma_i + \gamma_1} + \gamma_i \right) \right). \quad (19)$$

Proof: See Appendix B. ■

Remark 1: In Theorem 3, we assume that all the users want to transmit their data during each round of full data exchange. In the scenarios that some of the users have very poor channel conditions, one may be able to achieve higher sum rates than (19) by silencing weak users and forcing them to only listen. This could potentially allow for more channel utilization by the stronger users. Even for such scenarios, the results of Theorem 3 can be effectively used to find the set of the users that should remain active and their pairing which results in the maximum sum

rate of the system. We do this through running an exhaustive search where at each iteration, we force $i \in \{1, 2, \dots, N-1\}$ users to stay silent and find the maximum sum rate. At the end of this search, we are able to determine which users should remain silent to achieve the maximum sum rate. To find the complexity of this exhaustive search, one should note that to achieve possible rate improvement by silencing i users, it can be shown using (19) that these i users should be chosen from $S = \{U_1, U_2, \dots, U_i, U_{i+1}, U_N\}$. Checking all possibilities of choosing i users from S has a complexity of $O(N^2)$. For each choice of i users from S , the optimal ordering for the other $N - i$ active users is found using Theorem 3. Note that feasibility requires having $N - i$ uplink phases in this case so that the silent users can also decode the messages. The first $N - i - 1$ uplink phases can be assigned to the pairwise pairing of the $N - i$ transmitting users. In the last uplink phase, one of the users, say U_j , simply transmits its data to the relay. In the last downlink phase, the relay broadcasts X_j to all users. It can be easily seen that this extra uplink phase required for ensuring feasibility does not change the optimal pairing. Further, since

$$\log_2(1 + \gamma_j) > \log_2\left(\frac{\gamma_j}{\gamma_j + \gamma_k} + \gamma_j\right) \quad (20)$$

for any user k , the bound on the individual uplink transmission by U_j in the last phase is always larger than its uplink bound in a pairwise transmission. Thus, any of the $N - i$ active users can be selected to transmit individually in the last phase without affecting the sum rate. Here, when using (19) to calculate the sum rate, the pre-log factor in (19) for each subset of size $N - i$ should be $\frac{1}{2(N-i)}$ instead of $\frac{1}{2(N-1)}$. Each iteration of exhaustive search requires quadratic time complexity in i . Thus, the overall complexity of an exhaustive search over all values of i is $O(N^3)$, that is, polynomial in N .

Remark 2: From Theorem 3, one can show that the maximum achievable sum rate for the

optimal pairing is

$$R_s(G_O) = \frac{1}{2(N-1)} \times \log_2 \left(\left(\prod_{i=1}^N \gamma_i \right) \left(\prod_{i=2}^N 1 + \frac{1}{\gamma_i + \gamma_1} \right) \left(1 + \frac{1}{\gamma_1 + \gamma_N} \right) \right). \quad (21)$$

Thus, the maximum sum rate can be upper bounded by

$$R_s(G_O) \leq \frac{1}{2(N-1)} \log_2 \left(\prod_{i=1}^N \gamma_i \times \left(1 + \frac{1}{2\gamma_1} \right)^N \right). \quad (22)$$

Similarly, we can show that for a random pairing O' , the corresponding sum rate is lower bounded by

$$R_s(G_{O'}) \geq \frac{1}{2(N-1)} \log_2 \left(\prod_{i=1}^N \gamma_i \times \left(1 + \frac{1}{2\gamma_N} \right)^N \right). \quad (23)$$

According to (22) and (23), we find an upper bound for the difference between the sum rate of a random pairing and the optimal pairing as follows

$$R_s(G_O) - R_s(G_{O'}) \leq \frac{1}{2} \log_2 \left(\left(\frac{\gamma_N(1 + 2\gamma_1)}{\gamma_1(1 + 2\gamma_N)} \right)^N \right) \quad (24)$$

and as a result

$$\lim_{\gamma_1 \rightarrow \infty} (R_s(G_O) - R_s(G_{O'})) = 0. \quad (25)$$

Interestingly, (25) shows that for FDF relaying in high SNR regime, the performance of a randomly chosen pairing approaches the performance of the optimal pairing.

V. SIMULATION RESULTS

In this section, we investigate the performance of the optimal pairing in comparison with random pairings. pairing proposed in [14] is considered as well. We also compare our results with a transmission scheme in which users transmit their data in a time division multiplexing (TDM) fashion. For TDM considered here, uplink is divided into N equal in duration time slots and each user transmits in only one of them. Each uplink is then followed by a downlink in which

the relay broadcasts the received message to all users. For TDM, we account for the number of uplink and downlink time slots by considering a pre-log factor of $\frac{1}{2N}$ for the achievable rate of TDM. Further, the transmit power of each user is scaled such that each user has the same average transmit power as in the optimal pairing.

We use Monte Carlo simulation to average over common rate and sum rate for the optimal pairing and a randomly selected pairing. For each simulation round, random pairing is selected uniformly at random from all of the feasible client trees. For all users, P_i is set to 1. We assume that transmit power of the relay is proportional to the number of users in the system and set $P_r = N$. Channel gains are assumed to follow Rayleigh distribution and are amplitude samples of a circularly symmetric complex normal random variable $\mathcal{CN}(0, 1)$. The number of users is set to $N = 4$.

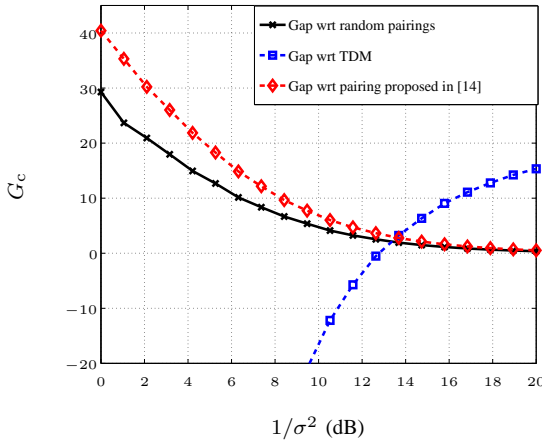


Fig. 4. Common rate gap wrt TDM transmission, random pairing, and pairing proposed in [14] for $N = 4$.

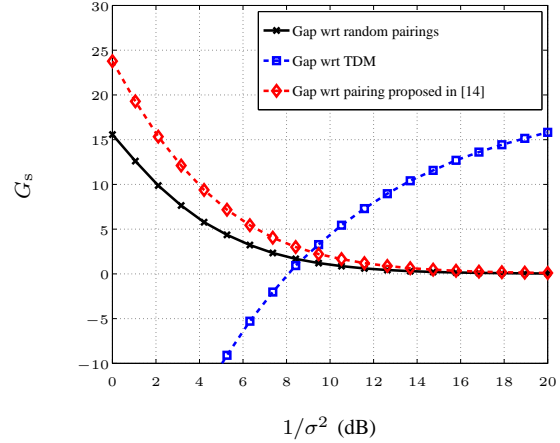


Fig. 5. Sum rate gap wrt TDM transmission, random pairing, and pairing proposed in [14] for $N = 4$.

In order to illustrate the difference between optimal pairing and other pairings, we define the *common rate gap* [7] of the the optimal pairing, O , with respect to (wrt) a given pairing, O' , as $G_c = 100 \times \frac{R_c(G_O) - R_c(G_{O'})}{R_c(G_O)}$ where, by abuse of notation, we denote the average of

common rate over all of the simulation rounds by $R_c(\cdot)$. Similarly, we define the *sum rate gap* as $G_s = 100 \times \frac{R_s(G_O) - R_s(G_{O'})}{R_s(G_O)}$.

Fig. 4 and 5 illustrate the aforementioned gap and feature the effect of optimal pairing on both common rate and sum rate for FDF relaying. Compared to random pairings, these figures show that the effect of pairing is not significant in higher SNR regimes, as proved earlier. The negative values of the gap function wrt TDM indicates that TDM performs better than pairwise transmission in the low SNR regime.

The importance of choosing the optimal pairing may vary when the number of users changes. Fig. 6 illustrates the performance of optimal pairing, random pairing, and pairing proposed in [14] in comparison with the cut-set bound (see [24], [1] and Appendix C for details.) for different numbers of users in a low SNR setting ($1/\sigma^2 = 5$ dB) and a high SNR setting ($1/\sigma^2 = 30$ dB). As seen, for smaller SNRs, the performance improvement of optimal pairing over the random pairing for FDF is more pronounced for larger N . This is because as N becomes larger, it is more likely to observe users with highly different channel qualities. This in turn signifies the importance of the pairing as it becomes more important to avoid rate-degrading pairs in the network.

VI. CONCLUSION

In this paper, we studied the effect of users' transmission pairing on the common rate and sum rate of the MWRC with pairwise transmissions and FDF relaying. Optimal pairings were found that maximize common rate and (under a mild practical assumption) sum rate in the system. Moreover, we showed that for high SNR regimes, the effect of pairing becomes less important. Our claims were supported and verified by computer simulations.

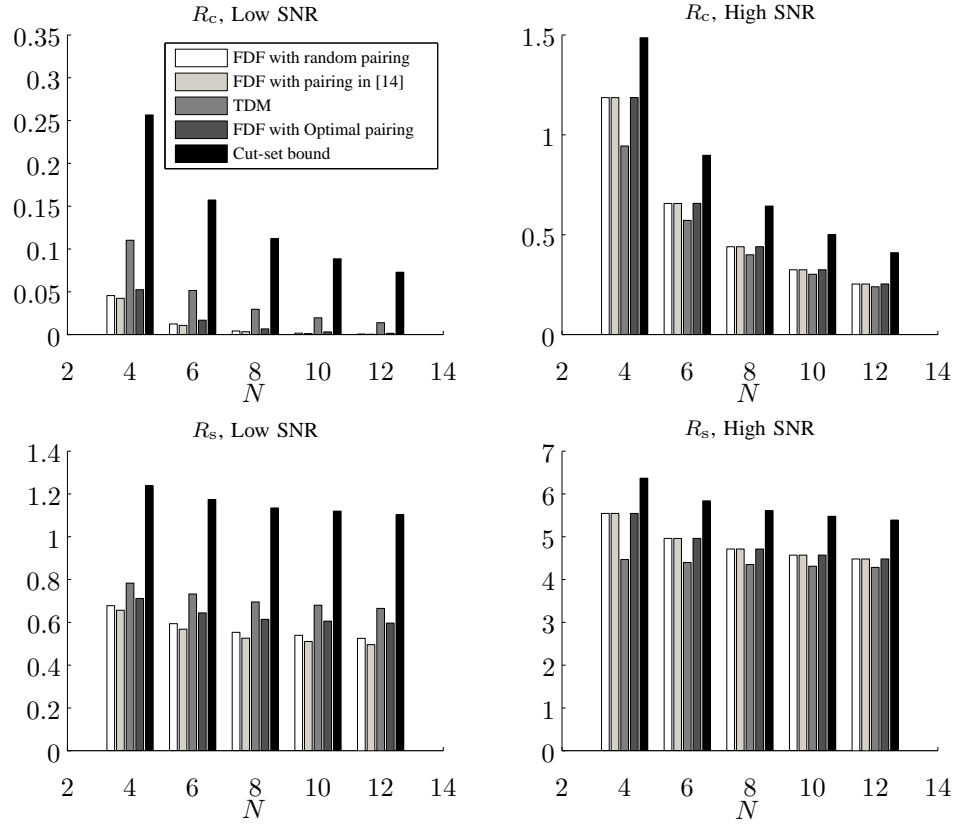


Fig. 6. Common rate and sum rate of the MWRC with pairwise transmissions and FDF relaying for different number of users, N .

APPENDIX A

PROOF OF THEOREM 2

Here, by an optimal tree, we mean a client tree that achieves the maximum R_c with respect to (8). There are two statements regarding (8) which we use to prove the theorem:

- 1) The function $f(x) = x \left(1 + \frac{1}{x+\alpha}\right)$, for $\alpha > 0$, is an increasing function of x .
- 2) The function $g(x) = \left(1 + \frac{1}{\alpha+x}\right)$ is a decreasing function of x .

Given a client tree, $G_O(V, E)$, with an FDF MWRC, we have

$$R_c(G_O) = \min_{i,j} \left\{ \frac{1}{2(N-1)} \log_2 \left(\gamma_i + \frac{\gamma_i}{\gamma_i + \gamma_j} \right) \right\}. \quad (26)$$

where $\gamma_i \leq \gamma_j$ and $v_i v_j \in E$. Using (26), we prove the following lemma.

Lemma 1. *There exists an optimal tree, $G_O(V, E)$, in which $A_1^{G_O} = \{v_2\}$.*

Proof: We adapt $G_{O'}(V, E')$ from G_O such that we disconnect all of the neighbors of v_1 from v_1 and connect them to v_2 . We also make v_1 and v_2 neighbors. More precisely,

$$E' = (E - \{v_1 v_i | v_i \in A_1^{G_O}\}) \cup \{v_2 v_i | v_i \in A_1^{G_O}; i \neq 2\} \cup \{v_1 v_2\} \quad (27)$$

Because of monotonicity of $f(x)$ and $g(x)$, to verify that $R_c(G_O) \leq R_c(G_{O'})$, we just need to show

$$\gamma_1 \left(1 + \frac{1}{\gamma_1 + \gamma_{\min}} \right) \leq \gamma_2 \left(1 + \frac{1}{\gamma_2 + \gamma_1} \right) \quad (28)$$

where, $\gamma_{\min} = \min\{\gamma_i | v_i \in A_1^{G_O}\}$. After some manipulation, we find that (28) is equivalent to

$$0 \leq (\gamma_2 - \gamma_1)(\gamma_1 + \gamma_{\min})(\gamma_2 + \gamma_1) + \gamma_2 \gamma_{\min} - \gamma_1^2 \quad (29)$$

which, according to the fact that $\gamma_1 \leq \gamma_{\min}$, is true. ■

We prove the theorem by induction. If $N = 2$ the theorem obviously holds. Now, assume that the statement of the theorem holds for every FDF MWRC with $N = k$. We show that it also holds for any FDF MWRC with $N = k + 1$. For $N = k + 1$, according to Lemma 1, there exists an optimal tree $G_O(V, E)$ in which $A_1^{G_O} = \{v_2\}$. From equation (26), we also have:

$$\begin{aligned} R_c(G_O) = & \min_{i,j} \left\{ \frac{1}{2(N-1)} \log_2 \left(\gamma_i + \frac{\gamma_i}{\gamma_i + \gamma_j} \right) \mid 1 < i \leq j; v_i v_j \in E \right\} \\ & \cup \left\{ \frac{1}{2(N-1)} \log_2 \left(\gamma_1 + \frac{\gamma_1}{\gamma_1 + \gamma_2} \right) \right\} \end{aligned} \quad (30)$$

If the second term in (30) is the limiting term in all of the possible client trees with $A_1^{G_O} = \{v_2\}$, the proposed pairing is optimal. Otherwise, maximizing $R_c(G_O)$ is equivalent to maximizing

$$\min \left\{ \gamma_i \left(1 + \frac{1}{\gamma_i + \gamma_j} \right) \mid 1 < i \leq j; v_i v_j \in E \right\}. \quad (31)$$

It is equivalent to maximizing the R_c for $G_{O'}(V', E')$, in which $V' = V - \{v_1\}$ and $E' = E - \{v_1 v_m \mid v_m \in A_1^{G_O}\}$. According to the induction hypothesis, it happens when

$$O' = \{\{v_2 v_3\}, \{v_3 v_4\}, \dots, \{v_{N-1} v_N\}\} \quad (32)$$

and as a result

$$O = \{\{v_1 v_2\}, \{v_2 v_3\}, \dots, \{v_{N-1} v_N\}\} \quad (33)$$

■

APPENDIX B

PROOF OF THEOREM 3

Here, we prove Theorem 3. To prove the theorem, we first show that there is an optimal tree with $\deg(v_N) = 1$ (Lemma 2). Then we prove that in the optimal tree each node needs to have only one neighbor among nodes with a lower SNR (Lemma 3). We then show that there exist an optimal tree with $\deg(v_N) = \deg(v_{N-1}) = 1$ (Lemma 4). In the next step, we prove that in an optimal tree for two nodes of degree one, say v_i and v_j , if v_i has a higher SNR than v_j then the neighbor of v_i has a higher SNR than the neighbor of v_j (Lemma 5). Then we prove the theorem by induction (Lemma 6).

Proof: We use the following convention for the rest of this proof:

$$d_i \triangleq 2^{2(N-1)R_i}. \quad (34)$$

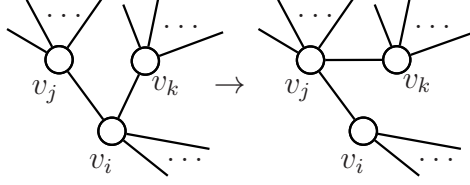


Fig. 7. Operation of \mathcal{V} -transform, $\mathcal{V}(G, v_i, v_j, v_k)$.

As a result, the bound given by (17) is equivalent to

$$d_i \leq \gamma_i \left(1 + \frac{1}{\gamma_i + \gamma_j} \right). \quad (35)$$

We also define $D_s(G_O) = \max \prod_{i=1}^N d_i = 2^{2(N-1)R_s(G_O)}$. Assume that $G(V, E)$ is a tree such that $\{v_i, v_j, v_k\} \subseteq V$ and $\{v_i v_j, v_i v_k\} \subseteq E$. We define a \mathcal{V} -transform on G in such a way that $\mathcal{V}(G, v_i, v_j, v_k) = G'(V, E')$ and $E' = (E - \{v_i v_k\}) \cup \{v_j v_k\}$. Fig. 7 shows the operation of a \mathcal{V} -transform.

Lemma 2. *There exists an optimal tree in which $\deg(v_N) = 1$.*

Proof: Assume G_O is an optimal tree in which $\deg(v_N) > 1$ and v_i and v_j are two neighbors of v_N and γ_j is the minimum SNR value of the neighbors of v_N . Consequently, we have $\gamma_i \geq \gamma_j$. It is straightforward to show that by performing a \mathcal{V} -transform on G_O and transform it to $G_{O'} = \mathcal{V}(G_O, v_N, v_i, v_j)$, we have $\frac{D_s(G_{O'})}{D_s(G_O)} \geq 1$:

$$\frac{D_s(G_{O'})}{D_s(G_O)} \geq \frac{\left(1 + \frac{1}{\gamma_N + \gamma_i}\right) \left(1 + \frac{1}{\gamma_i + h^{G_{O'}}(v_i)}\right) \left(1 + \frac{1}{\gamma_j + h^{G_{O'}}(v_j)}\right)}{\left(1 + \frac{1}{\gamma_i + \gamma_N}\right) \left(1 + \frac{1}{\gamma_i + \gamma_N}\right) \left(1 + \frac{1}{\gamma_j + \gamma_N}\right)} \geq 1. \quad (36)$$

Here, $h^{G_{O'}}(v_m)$ is the highest SNR of neighbors of v_m in $G_{O'}$. This shows that the sum rate of $G_{O'}$ is not less than sum rate of G_O . Note that, after applying this \mathcal{V} -transform, we have reduced degree of v_N by one. After applying $\deg(v_N) - 2$ more \mathcal{V} -transforms, we end up with an optimal tree with $\deg(v_N) = 1$. Fig. 8 illustrates a hypothetical optimal tree with $\deg(v_N) = 4$. It shows

how we apply 3 \mathcal{V} -transforms to get an optimal tree with $\deg(v_N) = 1$.

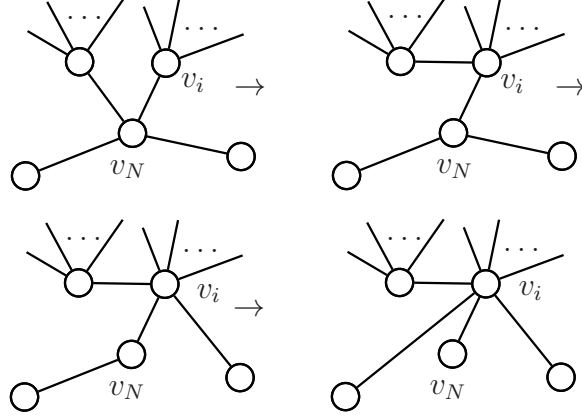


Fig. 8. Applying 3 \mathcal{V} -transform on an optimal tree with $\deg(v_N) = 4$.

■

Lemma 3. *There exists an optimal tree, $G_O(V, E)$, such that for any $0 < i < N-1$, $\deg(v_{N-i}) \leq i + 1$. Furthermore, the number of neighbors of v_{N-i} with a lower SNR than γ_{N-i} is at most one and consequently, the number of neighbors of v_{N-i} which have higher SNR than γ_{N-i} is at least $\deg(v_{N-i}) - 1$.*

Proof: If the number of those neighbors of v_{N-i} that have a lower SNR value than γ_{N-i} is a , after applying $(a - 1)$ \mathcal{V} -transforms, we end up with an optimal tree in which $\deg(v_{N-i}) \leq i + 1$. These $(a - 1)$ \mathcal{V} -transforms have the form $\mathcal{V}(G, V_{N-i}, v_i, v_k)$ and v_k has the highest SNR value among all of the neighbors of v_{N-i} .

Now, assume that $\deg(v_{N-i}) \leq i + 1$ and v_{N-i} has at most one neighbor v_j such that $j < N - i$. Then, we have that the number of neighbors of v_{N-i} that have a higher SNR than γ_{N-i} is greater than or equal to $|A_{N-i}^{G_O}| - 1 = \deg(v_{N-i}) - 1$. ■

Lemma 4. *There exists an optimal tree, $G_O(V, E)$, in which $\deg(v_N) = \deg(v_{N-1}) = 1$.*

Moreover, if v_j is the only neighbor of v_{N-1} and v_i is the only neighbor of v_N , then $\gamma_i \geq \gamma_j$.

Proof: If $\deg(v_{N-1}) = 2$, according to Lemma 3 and 2, there exists an optimal tree $G_O(V, E)$ in which $\deg(v_N) = 1$ and $v_N v_{N-1} \in E$. Let the other neighbor of v_{N-1} be v_j . Then, $G_{O'} = \mathcal{V}(G_O, v_N, v_i, v_j)$ is an optimal tree in which $\deg(v_{N-1}) = 1$. So, there always exists an optimal tree G_O , with $\deg(v_N) = \deg(v_{N-1}) = 1$. Now assume that $\deg(v_{N-1}) = 1$ and the only neighbor of v_{N-1} is v_j . If $v_j = v_N$, the graph will be disconnected. Otherwise, if the only neighbor of v_N is v_i , we want to prove that $\gamma_i \geq \gamma_j$. We also assume $\gamma_N \neq \gamma_{N-1}$; otherwise, one can rename the nodes in such a way that theorem holds. Assume that $G_{O''}(V, E'')$ is a client tree in which:

$$E'' = (E - \{v_N v_i, v_{N-1} v_j\}) \cup \{v_N v_j, v_{N-1} v_i\}. \quad (37)$$

We show that $D_s(G_{O''}) \leq D_s(G_O)$ iff $\gamma_i \geq \gamma_j$:

$$\frac{D_s(G_{O''})}{D_s(G_O)} = \frac{\left(1 + \frac{1}{\gamma_N + \gamma_j}\right)^2 \left(1 + \frac{1}{\gamma_{N-1} + \gamma_i}\right)^2}{\left(1 + \frac{1}{\gamma_N + \gamma_i}\right)^2 \left(1 + \frac{1}{\gamma_{N-1} + \gamma_j}\right)^2} \quad (38)$$

and as a result:

$$\begin{aligned} \frac{D_s(G_{O''})}{D_s(G_O)} &\leq 1 \\ \Leftrightarrow \left(1 + \frac{1}{\gamma_N + \gamma_j}\right) \left(1 + \frac{1}{\gamma_{N-1} + \gamma_i}\right) &\leq \left(1 + \frac{1}{\gamma_N + \gamma_i}\right) \left(1 + \frac{1}{\gamma_{N-1} + \gamma_j}\right) \\ \Leftrightarrow \gamma_N \gamma_j + \gamma_i \gamma_{N-1} &\leq \gamma_N \gamma_i + \gamma_{N-1} \gamma_j \\ \Leftrightarrow \gamma_j &\leq \gamma_i. \end{aligned}$$

■

Next lemma, is a generalization of Lemma 4 and we prove it in a similar way.

Lemma 5. Assume that $G_O(V, E)$ is an optimal tree in which $\deg(v_N) = \deg(v_{N-1}) = \dots = \deg(v_{N-i}) = 1$ and $i < N - 1$. Also, assume that $q < p \leq i$ and $\{v_j v_{N-p}, v_k v_{N-q}\} \in E$. Then $\gamma_j \leq \gamma_k$.

Proof: It is obvious that $j > N - i$ and $k > N - i$, otherwise the graph is disconnected. Now, if $\gamma_k < \gamma_j$, according to Lemma 4, the graph $G_{O'}(V, E')$ with $E' = (E - \{v_j v_{N-p}, v_k v_{N-q}\}) \cup \{v_j v_{N-q}, v_k v_{N-p}\}$ has a greater sum rate which contradicts the fact that G_O is optimal. ■

Lemma 6. Assume $G_O(V, E)$ is an optimal tree and i is the largest integer such that

$$\deg(v_N) = \deg(v_{N-1}) = \dots = \deg(v_{N-i}) = 1. \quad (39)$$

If $i < N - 1$, then there exists an optimal tree $G_{O'}(V, E')$ in which

$$\deg(v_N) = \deg(v_{N-1}) = \dots = \deg(v_{N-i+1}) = 1. \quad (40)$$

Proof: Assume that $A_{N-i+1}^{G_O} \cap \{v_N, v_{N-1}, \dots, v_{N-i}\} = \{v_{m_1}, v_{m_2}, \dots, v_{m_n}\}$ where $m_1 > m_2 > \dots > m_n$. Define

$$B = A_{N-i+1}^{G_O} - \{v_N, v_{N-1}, \dots, v_{N-i}\}. \quad (41)$$

According to Lemma 3, we assume that $|B| \leq 1$. If $|B| = 0$, G_O is disconnected. Assume $B = \{v_j\}$. Consider $G_{O'}(V, E')$ such that

$$\begin{aligned} E' = & (E - \{v_{m_1} v_{N-i+1}, v_{m_2} v_{N-i+1}, \dots, v_{m_n} v_{N-i+1}\}) \\ & \cup \{v_{m_1} v_j, v_{m_2} v_j, \dots, v_{m_n} v_j\}. \end{aligned} \quad (42)$$

Then, one can conclude that $\frac{D_s(G_O)}{D_s(G_{O'})} \geq 1$ as follows:

$$\frac{D_s(G_O)}{D_s(G_{O'})} \geq \frac{\left(1 + \frac{1}{\gamma_{N-i+1} + \gamma_{m_1}}\right) \left(1 + \frac{1}{\gamma_j + h^{G_O}(v_j)}\right)}{\left(1 + \frac{1}{\gamma_{N-i+1} + \gamma_j}\right) \left(1 + \frac{1}{\gamma_j + h^{G_{O'}}(v_j)}\right)} \quad (43)$$

$$\Rightarrow \frac{D_s(G_O)}{D_s(G_{O'})} \geq \frac{\left(1 + \frac{1}{\gamma_{N-i+1} + \gamma_{m_1}}\right)}{\left(1 + \frac{1}{\gamma_{N-i+1} + \gamma_j}\right)} \geq 1. \quad (44)$$

■

According to Lemma 6, there exists an optimal tree with respect to (17) in which

$$\deg(v_N) = \deg(v_{N-1}) = \dots = \deg(v_2) = 1. \quad (45)$$

As a result, O is an optimal solution with respect to (17). The maximum achievable sum rate, $R_s(G_O)$, could be found directly from (19). ■

APPENDIX C

CUTSET BOUNDS

Here, we provide the upper bounds we used In Section V. For common rate, as it is shown in [7, Theorem 1], the common rate for AWGN channels is upper bounded by

$$R_c \leq \frac{1}{2(N-1)} \min \left\{ \log_2 \left(1 + \sum_{i=1}^{N-1} \gamma'_i \right), \log_2 (1 + \Gamma_d) \right\}. \quad (46)$$

This can be found by considering the cut set bound for the cut separating $\{U_2, U_3, \dots, U_N\}$ and the relay in the uplink and also the cut separating U_1 and the relay in the downlink. Note that γ'_i is the scaled uplink SNR for user U_i such that U_i has the same average transmit power as in the optimal pairing.

For sum rate, we consider the cut separating all users but U_i from the relay for the uplink and also the cut separating U_i from the relay for the downlink. Then we have

$$\sum_{j \neq i} R_j \leq \min \left\{ \frac{1}{2} \log_2 \left(1 + \sum_{j \neq i} \gamma'_j \right), \frac{1}{2} \log_2 (1 + \Gamma_i) \right\}. \quad (47)$$

Let $S_i = \min \left\{ \frac{1}{2} \log_2 \left(1 + \sum_{j \neq i} \gamma'_j \right), \frac{1}{2} \log_2 (1 + \Gamma_i) \right\}$. Then, by summing over (47) for all $i = 1, 2, \dots, N$ we get

$$\sum_{j=1}^N R_j \leq \frac{1}{N-1} \sum_{j=1}^N S_j. \quad (48)$$

We used (48) as the upper bound for sum rate in our simulation results.

ACKNOWLEDGMENT

The study presented in this paper is supported by TELUS Corporation and Natural Sciences and Engineering Research Council of Canada (NSERC).

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